**647510** **MSC2MatC204x** **Seat No :________**

# M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination

## March/April - 2024 (NEW COURSE)

## Methods in Partial Differential Equations (Core)

**Time: 02:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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**Q.1.(A) Answer any two questions out of three.** **(10)**

(1) Solve : $\frac{dx}{y+\alpha z} = \frac{dy}{z+\beta x} = \frac{dz}{x+\gamma y}$.

(2) State and prove the necessary and sufficient condition for a Pfaffian differential equation $X \cdot dr = 0$ is integrable.

(3) Verify that the equation $yz(y-z)dx + xz(x-z)dy + xy(x-y)dz = 0$ is integrable and find its solution.

**Q.1.(B) Answer any one question out of two.** **(04)**

(1) Find the orthogonal trajectories on the cone $x^2 - y^2 = z^2 tanx$ of its intersenction with the family of planes parallel to $z = 0$.

(2) Show that $(x^2z - y^3)dx - 3xy^2dy + x^3dz = 0$ is integrable and find its solution.

**Q.2.(A) Answer any two questions out of three.** **(10)**

(1) Prove that the general solution of the linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$, where $F$ is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ form a solution of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

(2) Let $u_i(x_1, x_2, \dots, x_n, z) = c_i \,; i = 1,2, \dots, n$ are independent solution of the equation $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \cdots = \frac{dx_n}{P_n} = \frac{dz}{R}$, then prove that the relation $\phi(u_1, u_2, \dots, u_n) = 0$, in which the function $\phi$ is arbitrary , is a general solution of the linear partial differential equation
$$P_1 \frac{\partial z}{\partial x_1} + P_2 \frac{\partial z}{\partial x_2} + \cdots + P_n \frac{\partial z}{\partial x_n} = R.$$

(3) Find the general integral of the linear partial differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$.

**Q.2.(B) Answer any one question out of two.** **(04)**

(1) Find the general integral of the differential equation
$$x^2 \frac{\partial z}{\partial x} - y^2 \frac{\partial z}{\partial y} = (x + y)z \,.$$

(2) Find the integral surface of the differential equation $x(y^2 - z)p - y(x^2 - z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

**Q.3.(A) Answer any two questions out of three.** **(10)**

(1) Find the solution of the equation $z = \frac{1}{2}(p^2 - q^2) + (p - x)(q - y)$ which passes through the $X$-axis.

(2) Find a complete integral of the equation $p^2z^2 + q^2 = 1$.

(3) Find a complete integral of $z^2(p^2z^2 + q^2) = 1$.

**Q.3.(B) Answer any one question out of two. (04)**

(1) Prove that if a characteristic strip contains at least one integral element of $F(x,y,z,p,q)=0$, it is an integral strip of the equation $F\left(x,y,z,z_x,z_y\right)=0$.

(2) Find a complete integral of the equation $p^2x-q^2y=z$.

**Q.4.(A) Answer any two questions out of three. (10)**

(1) Let $u_1,u_2,\dots,u_n$ be the solutions of the homogeneous linear partial differential equation $F(D,D')z=0$, then prove that $\sum_{r=1}^{n} c_r u_r$, where $c_r$'s are arbitrary constants, is also a solution.

(2) Solve $\left(\frac{y^2z}{x}\right)p+xzq=y^2$.

(3) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$

**Q.4.(B) Answer any one question out of two. (04)**

(1) Reduce the equation $\frac{\partial^2 z}{\partial x^2}-x^2\frac{\partial^2 z}{\partial y^2}=0$ to a canonical form.

(2) Find a particular integral of the equation $(D^2-D')z=2y-x^2$.

**Q.5.(A) Answer any two questions out of three. (10)**

(1) Find the integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ which contains the straight line $x+y=0, z=1$.

(2) Let $\beta_r D'+\gamma_r$ is a factor of $F(D,D')$ and $\phi(\xi)$ is an arbitrary function of the single variable $\xi$, then prove that if $\beta_r\neq 0$, $u_r=\exp\left(-\frac{\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$ is a solution of the equation $F(D,D')z=0$.

(3) Find a complete integral of $z=px+qy+p^2+q^2$.

**Q.5.(B) Answer any one question out of two. (04)**

(1) Solve $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=sinx$.

(2) Find a complete integral of $p_1^3+p_2^2+p_3=1$

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